

$$1.) \int x \cos(x^2) dx \stackrel{u=x^2}{=} \frac{1}{2} \int \cos(u) du \quad (1P)$$

$$= \frac{1}{2} \sin(x^2) + C_1 \quad (1P)$$

$$2.) \int \frac{dy}{y} = \int \frac{x}{x-1} dx$$

$$\ln|y| = x + \ln|x-1| + C_1 \quad (1P)$$

$$y = D(x-1)e^x, \quad 2 = D \cdot 1 \Rightarrow D = -2 \quad (1P)$$

$$3.) u(x) = e^{-x}, \quad u(x)y' = \int 2x e^{2x} u(x) dx \quad (1P)$$

$$= 2(x-1)e^x + C_1$$

$$\Rightarrow y = C_1 e^x + 2(x-1)e^{2x}, \quad 3 = C - 2 \Rightarrow C_1 = 5 \quad (1P)$$

$$4.) \frac{\partial(2xy^2 + 4x^3)}{\partial y} = 4xy = \frac{\partial(2(x^2+1)y)}{\partial x} \quad (1P)$$

$$\frac{\partial F}{\partial x} = 2xy^2 + 4x^3 \Rightarrow F = x^2 y^2 + x^4 + k(y)$$

$$\frac{\partial F}{\partial y} = 2x^2 y + 2y = 2x^2 y + k'(y) \Rightarrow k(y) = y^2 + C_1$$

$$y^2(1+x^2) + x^4 = C_1 \quad (1P)$$

$$5.) 6\lambda^2 + 5\lambda - 6 = (2\lambda+3)(3\lambda-2) = 0$$

$$\lambda^{(A)} = -\frac{3}{2}, \quad \lambda^{(B)} = \frac{2}{3}$$

$$y = C^{(A)} e^{-\frac{3}{2}x} + C^{(B)} e^{\frac{2}{3}x}$$

(1P)

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$$X) \quad g(x) = f(f(f(f(f(x)))))) \text{ is an even polynomial since } f(x) = 1-2x^2 \text{ is even} \Rightarrow g'(0) = 0 \quad (10P)$$

Alternative: By successive application of the chain rule

$$g'(0) = f'(f(f(f(f(0)))))) \cdot f'(f(f(f(0)))) \cdot$$

$$\cdot \underbrace{f'(f(f(0))) \cdot f'(f(0)) \cdot f'(0)}_{=0} = 0 \quad (10P)$$